The Vector Form of Coulomb's Law of Force

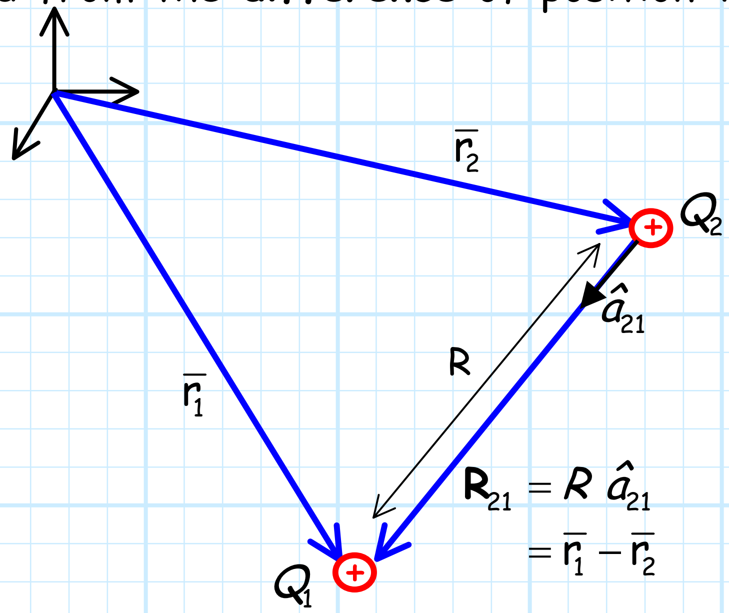
The **position vector** can be used to make the **calculations** of Coulomb's Law of Force more **explicit**. Recall:

$$\mathbf{F}_1 = \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\,\hat{a}_{21} \quad [N]$$

Specifically, we ask ourselves the question: **how** do we determine the **unit vector** $\hat{a}_{21}$ and **distance** R??

* Recall the **unit** vector $\hat{a}_{21}$ is a unit vector directed **from** Q_2 **toward** Q_1, and R is the **distance** between the two charges.

* The **directed distance** vector $\mathbf{R}_{21} = R\,\hat{a}_{21}$ can be determined from the **difference** of position vectors $\bar{r}_1$ and $\bar{r}_2$.



This directed distance $\mathbf{R}_{21} = \bar{r}_1 - \bar{r}_2$ is **all** we need to determine **both** unit vector $\hat{a}_{21}$ and distance R (i.e., $\mathbf{R}_{21} = R\,\hat{a}_{21}$)!

For example, since the **direction** of directed distance $\mathbf{R}_{21}$ is equal to $\hat{a}_{21}$, we can **explicitly** find this unit vector by **dividing** $\mathbf{R}_{21}$ by its **magnitude**:

$$\hat{a}_{21} = \frac{\mathbf{R}_{21}}{|\mathbf{R}_{21}|} = \frac{\bar{r}_1 - \bar{r}_2}{|\bar{r}_1 - \bar{r}_2|}$$

Likewise, the **distance** R between the two charges is simply the magnitude of directed distance $\mathbf{R}_{21}$!

$$R = |\mathbf{R}_{21}| = |\bar{r}_1 - \bar{r}_2|$$

Using these expressions, we find that we can express **Coulomb's Law** entirely in terms of $\mathbf{R}_{21}$, the **directed distance** relating the location of Q_1 with respect to Q_2:

$$\begin{aligned}\mathbf{F}_1 &= \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{R^2}\,\hat{a}_{21} \\ &= \frac{1}{4\pi\varepsilon_0}\frac{Q_1 Q_2}{|\mathbf{R}_{21}|^2}\,\frac{\mathbf{R}_{21}}{|\mathbf{R}_{21}|} \\ &= \frac{Q_1 Q_2}{4\pi\varepsilon_0}\,\frac{\mathbf{R}_{21}}{|\mathbf{R}_{21}|^3}\end{aligned}$$

Explicitly using the relation $\mathbf{R}_{21} = \bar{r}_1 - \bar{r}_2$, we find:

$$\mathbf{F}_1 = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{\mathbf{R}_{21}}{|\mathbf{R}_{21}|^3} = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{\bar{r}_1 - \bar{r}_2}{|\bar{r}_1 - \bar{r}_2|^3}$$

We of course could likewise define a directed distance:

$$\mathbf{R}_{12} = \bar{r}_2 - \bar{r}_1$$

which relates the location of Q_2 with respect to Q_1.

We can thus describe the force on charge Q_2 as:

$$\mathbf{F}_2 = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{\mathbf{R}_{12}}{|\mathbf{R}_{12}|^3} = \frac{Q_1 Q_2}{4\pi\varepsilon_0} \frac{\bar{r}_2 - \bar{r}_1}{|\bar{r}_2 - \bar{r}_1|^3}$$

Note since $\mathbf{R}_{12} = -\mathbf{R}_{21}$ (thus $|\mathbf{R}_{12}| = |\mathbf{R}_{21}|$), we again find that:

$$\mathbf{F}_2 = -\mathbf{F}_1$$

The forces on each charge have **equal** magnitude but **opposite** direction.

See Example 3-3 on pages 72-73 !